NEET | IIT-JEE | FOUNDATION

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JEE MAIN-2020

COMPUTER BASED TEST (CBT)

DATE : 04-09-2020 (SHIFT-2) | TIME : (3.00 pm to 6.00 pm)

Duration 3 Hours | Max. Marks : 300

QUESTION & SOLUTIONS

PART-A : PHYSICS

SECTION – 1 : (Maximum Marks : 80)

Single Choice Type

This section contains 20 Single choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

Full Marks : +4 If ONLY the correct option is chosen.

Negative Marks : –1 (minus one) mark will be deducted for indicating incorrect response.

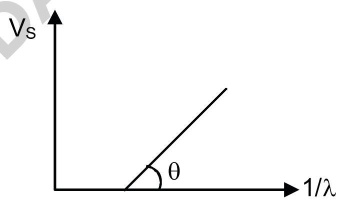
1. A body is moving in a low circular or it about a planet of mass M and radius R. The radius of the orbit can be taken to be R itself. Then the ratio of the speed of this body in the orbit to the escape velocity from the planet is :

(1) 1 (2) $\frac{1}{\sqrt{2}}$ (3) $\sqrt{2}$ (4) 2

Ans. (2)

Sol. $\frac{v_0}{v_e} = \frac{\sqrt{\frac{Gm}{r}}}{\sqrt{\frac{2Gm}{r}}} = \frac{1}{\sqrt{2}}$

2. In a photoelectric effect experiment, the graph of stopping potential V versus reciprocal of wavelength obtained is shown in the figure. As the intensity of incident radiation is increased :



(1) Slope of the straight line get more steep

(2) Graph does not change

(3) Straight line shifts to right

(4) Straight line shifts to left

Ans. (2)

Sol. $eV_S = h\nu - w$

$V_S = \frac{h\nu}{e} - \frac{w}{e}$

Frequency and work function are constant therefore graph does not change.

3. A small ball of mass m is thrown upward with velocity u from the ground. The ball experiences a resistive force mkv^2 where v is it speed. The maximum height attained by the ball is :

(1) $\frac{1}{2k}\tan^{-1}\frac{ku^2}{g}$ (2) $\frac{1}{k}\ell n\left(1+\frac{ku^2}{2g}\right)$ (3) $\frac{1}{k}\tan^{-1}\frac{ku^2}{2g}$ (4) $\frac{1}{2k}\ell n\left(1+\frac{ku^2}{g}\right)$

Ans. (4)

Sol. $F_{net} = ma$

$-mg - mkv^2 = mv\frac{dv}{ds}$

$v\frac{dv}{ds} = -g - kv^2$

$$-\int_{v_0}^{0}\frac{vdv}{g+kv^2}=\int_{0}^{h_{max}}ds=h_{max}$$

$$h_{max}=\frac{1}{2k}\ell n\left(\frac{g+kv_0^2}{g}\right)$$

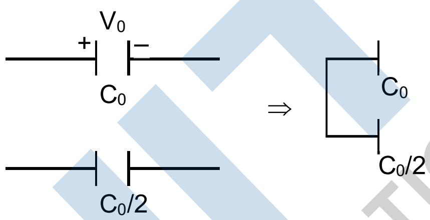
4. A capacitor C is fully charged with voltage V_0. After disconnecting the voltage source, it is connected in parallel with another uncharged capacitor of capacitance C/2. The energy loss in the process after the charge is distributed between the two capacitors is :

(1) $\frac{1}{6}CV_0^2$ (2) $\frac{1}{2}CV_0^2$ (3) $\frac{1}{4}CV_0^2$ (4) $\frac{1}{3}CV_0^2$

Ans. Correction answer is (1) but IIT gives (3)

Sol. heat loss

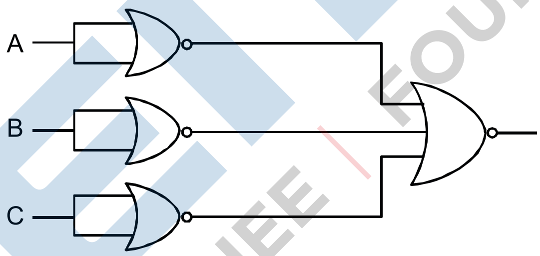
$$H=\frac{C_1C_2}{2(C_1+C_2)}(V_1-V_2)^2$$



$$=\frac{C\frac{C}{2}}{2\left(C+\frac{C}{2}\right)}(V_0-0)^2=\frac{C}{6}V_0^2$$

$$H=\frac{1}{6}C_0V_0^2$$

5. Identify the operation performed by the circuit given below :



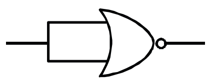
(1) NOT (2) AND (3) NAND (4) OR

Ans. (2)

Sol. Behaves like a not gate so boolen equation will be

$$y=\overline{\overline{A}+\overline{B}+\overline{C}}$$

$$y = A . B . C$$



whole arrangement behaves like a AND gate

6. The electric field of a plane electromagnetic wave is given by $\vec{E}=E_0(\hat{x}+\hat{y})\sin(kz-\omega t)$. Its magnetic field will be given by

(1) $\frac{E_0}{c}(\hat{x}+\hat{y})\sin(kz-\omega t)$ (2) $\frac{E_0}{c}(\hat{x}-\hat{y})\sin(kz-\omega t)$

(3) $\frac{E_0}{c}(\hat{x}-\hat{y})\cos(kz-\omega t)$ (4) $\frac{E_0}{c}(-\hat{x}-\hat{y})\sin(kz-\omega t)$

Ans. (4)

Sol. $\vec{E} \times \vec{B}$ □ $\vec{C}$

7. The driver of bus approaching a big wall notices that the frequency of his bus's horn changes from 420 Hz to 490 Hz when he hears it after it gets reflected from the wall. Find the speed of the bus if speed of the sound is 330 ms^{-1} :

(1) 81 kmh^{-1} (2) 71 kmh^{-1} (3) 61 kmh^{-1} (4) 91 kmh^{-1}

Ans. (4)

Sol. Frequency appeared at wall

$$f_w = \frac{330}{330 - v}.f \qquad ...(1)$$

$$f' = \frac{330 + v}{330}.f_w = \frac{330 + v}{330 - v}.f$$

$$490 = \frac{330 + v}{330 - v}.420$$

$$v = \frac{330 \times 7}{91} \approx 25.38 \text{ m/s} = 91 \text{ Km/s}$$

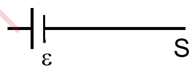
8. A series L-R circuit is connected to a battery of emf V. If the circuit is switched on at t = 0, then the time at which the energy stored in the inductor reaches (1/n) times of its maximum value, is :

(1) $\frac{L}{R}\ell n\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)$ (2) $\frac{L}{R}\ell n\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$ (3) $\frac{L}{R}\ell n\left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)$ (4) $\frac{L}{R}\ell n\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)$

Ans. (1)

Sol. Potential energy stored in inductor is given by $U = \frac{1}{2}LI^2$

$$U \propto I^2$$



$$\frac{U}{U_0} = \left(\frac{I}{I_0}\right)^2$$

$$\frac{1}{n} = \left(\frac{I}{I_0}\right)^2$$

$$\frac{I}{I_0} = 1 - e^{-RT/L} = \frac{1}{\sqrt{n}}$$

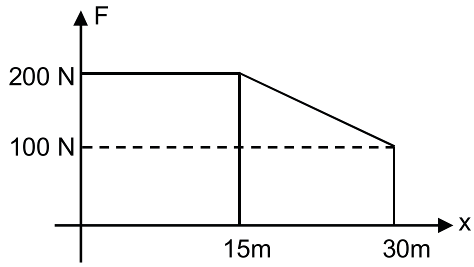
$$t = \frac{L}{R}\ln\frac{\sqrt{n}}{\sqrt{n}-1}$$

9. A person pushes a box on a rough horizontal platform surface. He applies a force of 200 N over a distance of 15m. Thereafter, he gets progressively tired and his applied force reduces linearly with distance to 100 N. The total distance through which the box has been moved is 30 m. What is the work done by the person during the total movement of the box?

(1) 5250 J (2) 3280 J (3) 2780 J (4) 5690 J

Ans. (1)

Sol.



$W = \text{area} = (200 \times 15) + \frac{1}{2}(100 + 200) \times 15$

$= 3000 + 2250$

$W = 5250$ J

10. Match the thermodynamics processes taking place in a system with the correct conditions. In the table: DQ is the heat supplied, DW is the work done and DU is change in internal energy of the system. Match the following

(I)	Adiabatic	(A)	$\Delta W = 0$
(II)	Isothermal	(B)	$\Delta Q = 0$
(III)	Isobaric	(C)	$\Delta U \neq 0, \Delta W \neq 0$ $\Delta Q \neq 0$
(IV)	Isochoric	(D)	$\Delta U = 0$

(1) $I \to A$	$II \to A$	$III \to B$	$IV \to C$
(2) $I \to B$	$II \to D$	$III \to A$	$IV \to C$
(3) $I \to A$	$II \to B$	$III \to D$	$IV \to D$
(4) $I \to B$	$II \to A$	$III \to D$	$IV \to Cs$

Ans. (2)

Sol. In Adiabatic $\Delta Q = 0$

In Isothermal $\Delta U = 0$

In Isochoric $\Delta W \neq 0$

11. Consider two uniform discs of the same thickness and different radii $R_1 = R$ and $R_2 = \alpha R$ made of the same material. If the ratio of their moments of inertia I_1 and I_2, respectively, about their axes is $I_1 : I_2 = 1 : 16$ then the value of α is :

(1) 2 (2) 4 (3) $2\sqrt{2}$ (4) $\sqrt{2}$

Ans. (1)

Sol. Moment of inertia of disc is given by $I = \frac{MR^2}{2} = \frac{[\rho(\pi R^2)t]R^2}{2}$

$I \propto R^4$

$\frac{I_2}{I_1} = \left(\frac{R_2}{R_1}\right)^4$

$\frac{16}{1} = \alpha^4$

$\alpha = 2$

12. A quantity x is given by $(1Fv^2/WL^4)$ in terms of moment of inertia I, force F, velocity v, work W and length L. The dimensional formula for x is same as that of :

(1) force constant (2) energy density (3) Planck's constant (4) coefficient of viscosity

Ans. (2)

Sol. $\frac{IFv^2}{W\ L^4} = \frac{(M^1L^2)(M^1L^1T^{-2})(L^1T^{-2})^2}{(M^1L^2T^{-2})(L^4)} = \frac{M^1L^{-2}T^{-2}}{L^3} = M^1L^{-1}T^{-2} = \text{Energy density}$

13. A circular coil has moment of inertia 0.8 kg m^2 around any diameter and is carrying current to produce a magnetic moment of 20 am^2. The coil is kept initially in a vertical position and it can rotate freely around a horizontal diameter. When a uniform magnetic field of 4T is applied along the vertical, it starts rotating around its horizontal diameter. The angular speed the coil acquires after rotating by 60° will be :

(1) 20 rad s^{-1} (2) 10 rad s^{-1} (3) 20π rad s^{-1} (4) 10π rad s^{-1}

Ans. (2)

Sol. From energy conservation

$$\frac{1}{2}I\omega^2 = U_{in} - U_f$$

$$= -MB\cos 60° - (-MB)$$

$$\frac{MB}{2} = \frac{1}{2}I\omega^2$$

$$\frac{20 \times 4}{2} = \frac{1}{2}(0.8)\omega^2$$

$100 = \omega^2$

$\omega = 10$ rad

14. A paramagnetic sample shows a net magnetisation of 6A/m when it is placed in an external magnetic field of 0.4 T at a temperature of 4K. When the sample is placed in an external magnetic field of 0.3T at a temperature of 24K, then the magnetisation will be :

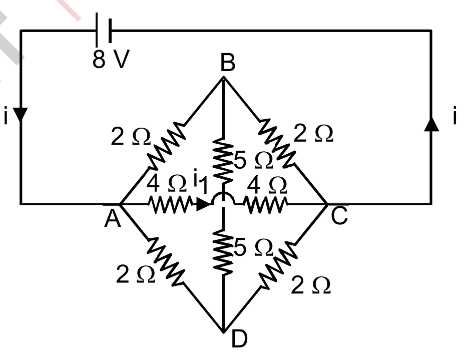
(1) 4 A/m (2) 0.75 A/m (3) 2.25 A/m (4) 1 A/m

Ans. (2)

Sol. $M = \frac{CB_{ext}}{T}$

Putting the value we get N = 0.25 A/m

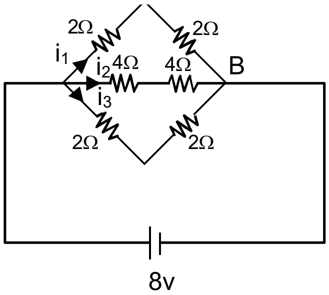
15. The value of current i1 flowing from A to C in the circuit diagram is :



(1) 5A (2) 0.75 A/m (3) 2.25 A/m (4) 1 A/m

Ans. (2)

Sol.



$i_2 = \frac{8}{4+4} = 1\,\text{Amp}$

16. A particle of charge q and mass m is subjected to an electric field $E = E_0(1–ax^2)$ in the x–direction, where a and E_0 are constants. Initially the particle was at rest at x = 0. Other than the initial position the kinetic energy of the particle becomes zero when the distance of the particle from the origin is :

(1) a (2) $\sqrt{\frac{1}{a}}$ (3) $\sqrt{\frac{3}{a}}$ (4) $\sqrt{\frac{2}{a}}$

Ans. (3)

Sol. $W_{ex} = \Delta K$ $K_f – K_i = 0$

$\int_0^x qE\,dx = 0$

$q\int_0^x E_0(1-Qx^2)dx = 0$

$qE_0\int_0^x (1-ax^2)dx = 0$

$x - \frac{ax^3}{3} = 0$

$1 - \frac{ax^2}{3} = 0$

$\frac{ax^2}{3} = 1$

$x^2 = \frac{3}{a}$

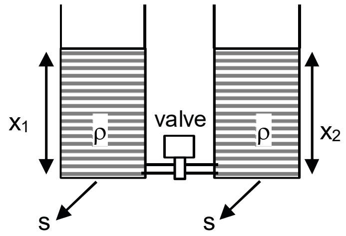
$x = \sqrt{\frac{3}{a}}$

17. Two identical cylindrical vessels are kept on the ground and each contain the same liquid of density d. The area of the base of both vessels is S but the height of liquid in one vessel is x_1 and in the other, x_2. When both cylinders are connected through a pipe of negligible volume very close to the bottom, the liquid flows from one vessel to the other until it comes to equilibrium at a new height. The change in energy of the system in the process is :

(1) $\frac{3}{4}gdS(x_2-x_1)^2$ (2) $\frac{1}{4}gdS(x_2-x_1)^2$ (3) $gdS(x_2+x_1)^2$ (4) $gdS(x_2^2+x_1^2)^2$

Ans. (2)

Sol.



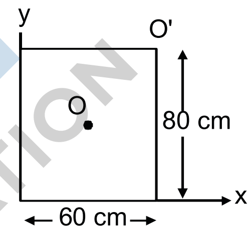
Initial height of liquid in container's of same cross section are x_1 and x_2 respectively. Now value is opened find loss in potential energy when water level be become same

loss in PE = $U_i - U_f$

$$= \left[\rho(A)x_1\frac{x_1}{2} + \rho(A)x_2\frac{x_2}{2}\right]g - \left[\rho A\left(\frac{x_1+x_2}{2}\right)\times\left(\frac{x_1+x_2}{4}\right)\times 2\right]g$$

$$= \rho Ag\left[\frac{x_1^2}{2} + \frac{x_2^2}{2} - \frac{(x_1+x_1)^2}{4}\right] = \frac{\rho Ag(x_1+x_2)^2}{4}$$

18. For a uniform rectangular sheet shown in the figure, the ratio of moments of inertia about the axes perpendicular to the sheet and passing through O (the centre of mass) and O' (corner point) is:



(1) 1/2 (2) 1/4

(3) 2/3 (4) 1/8

Ans. (2)

Sol. $$\frac{I_O}{I_{O'}} = \frac{\frac{M}{12}(a^2+b^2)}{\frac{M}{12}(a^2+b^2)+M\left(\frac{a^2}{4}+\frac{b^2}{4}\right)} = \frac{\frac{M}{12}(a^2+b^2)}{\frac{M}{3}(a^2+b^2)} = \frac{1}{4}$$

19. A cube of metal is subjected to a hydrostatic pressure 4GPa. The percentage change in the length of the side of the cube is close to : (Given bulk modulus of metal, B = 8 × 10^{10} Pa)

(1) 1.67 (2) 5 (3) 20 (4) 0.6

Ans. (1)

Sol. $$\Delta P = (B)\frac{\Delta V}{V} = B\times 3\frac{\Delta L}{L}$$

Putting the value of ΔP and B we get $\frac{\Delta L}{L}\times 100 = 1.67$

20. Find the Binding energy per nucleon for $^{50}_{120}Sn$. Mass of proton m_p = 1.00783 U, mass of neutron m_n = 1.00867 U and mass of tin nucleus m_{sn} = 119.902199 U. (take 1U = 931 MeV)

(1) 9.0 MeV (2) 8.5 MeV (3) 8.0 MeV (4) 7.5 MeV

Ans. (2)

Sol. Binding energy = $(\Delta M)\ C^2 = (\Delta M)\ 931$

put the value of ΔM

BE = 8.5 MeV

SECTION – 2 : (Maximum Marks : 20)

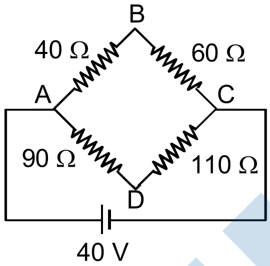
This section contains FIVE (05) questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.

If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Full Marks : +4 If ONLY the correct option is chosen.

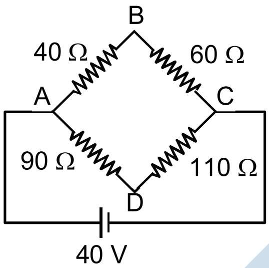
Zero Marks : 0 In all other cases

21. Four resistance 40Ω, 60Ω, 90Ω, and 110Ω make the arms of a quadrilateral ABCD. Across AC is a battery of emf 40V and internal resistance negligible. The potential difference across BD in V is ……



Ans. 2

Sol.



From KVL

$$V_B - 60\left(\frac{40}{100}\right) + 110\left(\frac{40}{200}\right) = V_D$$

$V_B - V_D = 2$

22. The distance between an object and a screen is 100 cm. A lens can produce real image of the object on the screen for two different positions between the screen and the object. The distance between these two positions is 40 cm. If the power of the lens is close to $\left(\frac{N}{100}\right)D$ where N is an integer, the value of N is ……….

Ans. 476.19

Note : NTA Answer is 5.

Sol. $f = \frac{D^2 - d^2}{4D} = \frac{100^2 - 40^2}{4(100)} = \frac{(100+40)(100-40)}{4(100)} = 21\,\text{cm}$

$P = \frac{1}{f} = \frac{100}{21} = \frac{N}{100}$

N = 476.19.

23. The change in the magnitude of the volume of an ideal gas when a small additional pressure ΔP is applied at a constant temperature, is the same as the change when the temperature is reduced by a small quantity ΔT at constant pressure. The initial temperature and pressure of the gas were 300 K and 2 atm. respectively. If $|\Delta T| = C|\Delta P|$ then value of C in (K/atm) is ….

Ans. 150

Sol. $PV = nRT$

$P\Delta V + V\Delta P = 0$

$\Delta V = -\frac{\Delta P}{P}V$ …(i)

In second case

$P\Delta V = -nR\Delta T$

$\Delta V = -\frac{nR\Delta T}{P}$ …(ii)

equating (i) and (ii)

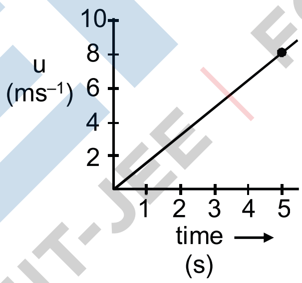
$\frac{nR\Delta T}{P} = \frac{\Delta P}{P}V$

$\Delta T = \Delta P\frac{V}{nR}$

$C = \frac{V}{nR}$

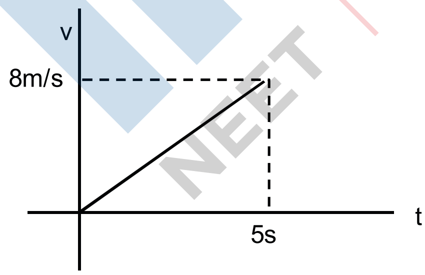
Putting the value of V, n and R, C = 150

24. The speed verses time graph for a particle is shown in the figure. The distance travelled (in m) by the particle during the time interval t = 0 to t = 5 s will be …..



Ans. 20.00

Sol.



Distance = Area of |v| – t graph

$= 1/2 \times 8 \times 5 = 20$ m

25. Orange light of wavelength 6000×10^{-10} m illuminates a single slit of width 0.6×10^{-4} m. the maximum possible number of diffraction minima produced on both sides of the central maximum is :….

Ans. 200

Sol. Light of wavelength 6000×10^{-10} m passes through a single slit of width 0.6×10^{-4} m. Find height of highest order of minima on both side central maxima

for minima

$d \sin\theta = n\lambda$

$$\sin\theta = \frac{n\lambda}{d} < 1$$

$$n \le \frac{d}{\lambda}$$

$$n < \frac{0.6 \times 10^{-4}}{6000 \times 10^{-10}}$$

$n < 100$

The total number of maxima of both side at central maxima = 100 + 100 = 200

PART-B : CHEMISTRY

SECTION – 1 : (Maximum Marks : 80)

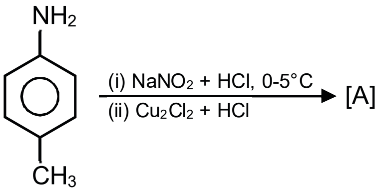
Single Choice Type

This section contains 20 Single choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

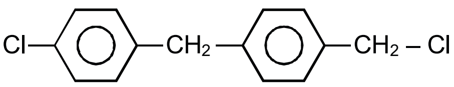
Full Marks : +4 If ONLY the correct option is chosen.

Negative Marks : –1 (minus one) mark will be deducted for indicating incorrect response.

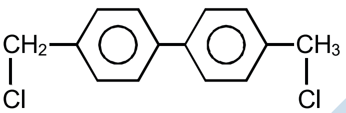
26. In the following reaction sequence, [C] is :

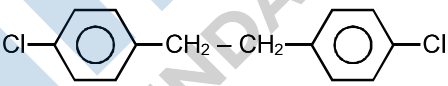


$\xrightarrow[h\nu]{Cl_2}$ [B] $\xrightarrow{\text{Na + dry ether}}$ [C]
(Major product)

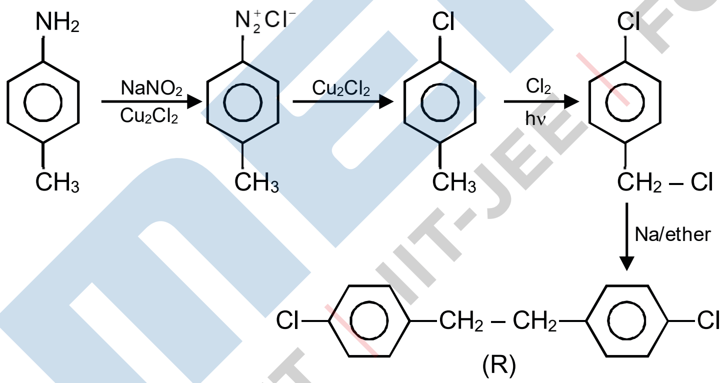
(1) 

(2) 

(3) 

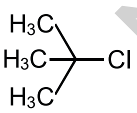
(4) 

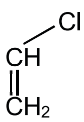
Ans. (4)

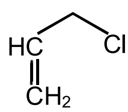
Sol. 

(R)
Final product

27. Among the following compounds, which one has the shortest C–Cl bond ?

(1) 

(2) 

(3) 

(4) $H_3C–Cl$

Ans. (2)

Sol. Due to conjugation of lone pair of chlorine with π bond of C–C, partial double bond character decrease bond length that's why $CH_2=CH–Cl$ have shortest C–Cl bond length.

28. If the equilibrium constant for $A \rightleftharpoons B + C$ is $K_{eq}^{(1)}$ and that of $B + C \rightleftharpoons P$ is $K_{eq}^{(2)}$, the equilibrium constant for A P is :

(1) $K_{eq}^{(1)} / K_{eq}^{(2)}$ (2) $K_{eq}^{(1)} + K_{eq}^{(2)}$ (3) $K_{eq}^{(2)} - K_{eq}^{(1)}$ (4) $K_{eq}^{(1)} K_{eq}^{(2)}$

Ans. (4)

Sol. On adding Reaction 1st and Reaction 2nd we get.

$A \rightleftharpoons P$ $K_{eq} = K_{eq}(1) . K_{eq}(2)$

29. The molecule in which hybrid MOs involve only one d-robital of the central atom is :

(1) $[Ni(CN)_4]^{2-}$ (2) XeF_4 (3) $[CrF_6]^{3-}$ (4) BrF_5

Ans. (1)

Sol.

Complex	Hybridisation
(1) $[Ni(CN)_4]^{2-}$	dsp^2
(2) XeF_4	$sp^3 d^2$
(3) $[CrF_6]^{3-}$	$sp^3 d^2$
(4) BrF_5	$sp^3 d^2$

30. The one that can exhibit highest paramagnetic behaviour among the following is :

gly = glycinato ; bpy = 2, 2'-bipyridine

(1) $[Fe(en)(bpy)(NH_3)_2]^{2+}$ (2) $[Pd(gly)_2]$

(3) $[Ti(NH_3)_6]^{3+}$ (4) $[Co(OX)_2(OH)_2]^-$ ($\Delta_0 > P$)

Ans. (4)

Sol.

Complex	EC	Unpaired electrons
(1) $[Fe(en)(bpy)(NH_3)_2]^{2+}$	$Fe^{2+} = 3d^6 = t_{2g}^{2,2,2}, eg^{0,0}$	0
(2) $[Pd(gly)_2]$	$Pd^{2+} = 4d^8$	0
(3) $[Ti(NH_3)_6]^{3+}$	$Ti^{3+} = 3d^1$	1
(4) $[Co(OX)_2(OH)_2]^-$ ($\Delta_0 > P$)	$Co^{5+} \Rightarrow 3d^4 \Rightarrow t_{2g}^{2,1,1}, eg^{0,0}$	2

31. The incorrect statement(s) among (a) - (c) is (are) :

(a) W(VI) is more stable than Cr(VI).

(b) in the presence of HCl, permanganate titrations provide satisfactory results.

(c) some lanthanoid oxides can be used as phosphors.

(1) (a) only (2) (b) and (c) only (3) (b) only (4) (a) and (b) only

Ans. (3)

Sol. (a) In transition metals on moving down the group higher oxidation states are more stable due to smaller size of atoms, due to lanthanide and actinide contractions.

(b) $KMnO_4$ can oxidise chloride into chlorine, so it will give incorrect results

(c) its a fact

32. The reaction in which the hybridisation of the underlined atom is affected is :

(1) $H_2\underline{S}O_4 + NaCl \xrightarrow{420\,K}$ (2) $\underline{N}H_3 \xrightarrow{H^+}$

(3) $H_3\underline{P}O_2 \xrightarrow{\text{Disproport ionation}}$ (4) $XeF_4 + SbF_5 \longrightarrow$

Ans. (4)

Sol. (1) $\underset{sp^3}{H_2SO_4} + NaCl \longrightarrow \underset{sp^3}{Na_2SO_4} + 2HCl$

(2) $\underset{sp^3}{NH_3} + H^+ \longrightarrow$ H–N(H)(H):→H^+ (sp³)

(3) $\underset{sp^3}{H_3PO_2} \longrightarrow \underset{sp^3}{H_3PO_4} + PH_3$

(4) $\underset{sp^3d^2}{XeF_4} + SbF_5 \longrightarrow \underset{sp^3d}{[XeF_3]^+}[SbF_6]^-$

33. An alkaline earth metal 'M' readily forms water soluble sulphate and water insoluble hydroxide. Its oxide MO is very stable to heat and does not have rock-salt structure. M is :

(1) Sr (2) Mg (3) Ca (4) Be

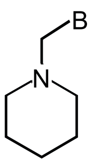
Ans. (4)

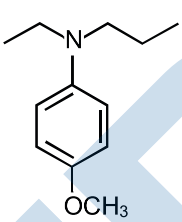
Sol. $BeSO_4$ Soluble in water

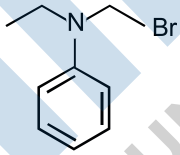
$Be(OH)_2$ Insoluble in water

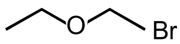
Structure of BeO is Hexagonal Wurtzite.

34. Which of the following compounds will form the precipitate with aq. $AgNO_3$ solution most readily ?

(1) 

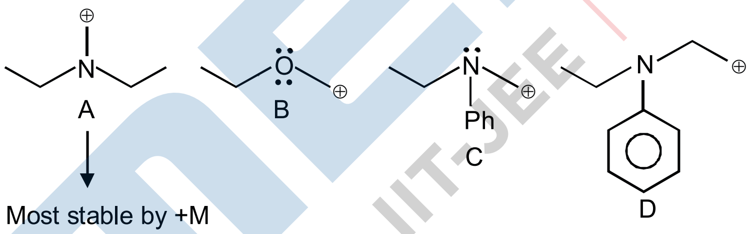
(2) 

(3) 

(4) 

Ans. (1)

Sol. Given reaction is an examples of S_N1 reaction. Which depend upon stability of carbocation.



Most stable by +M effect of –N̈< (amine)

35. The major product [B] in the following reaction is :

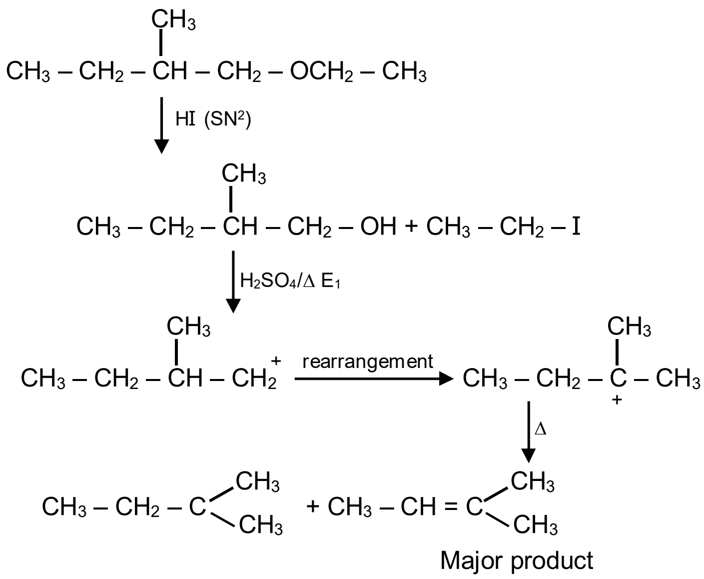
$$CH_3-CH_2-\underset{}{\overset{CH_3}{\overset{|}{C}H}}-CH_2-OCH_2-CH_3 \xrightarrow[\text{Heat}]{HI} [A]\ \text{alcohol} \xrightarrow[\Delta]{H_2SO_4} [B]$$

(1) $CH_2=CH_2$ (2) $CH_3-CH=\overset{CH_3}{\overset{|}{C}}-CH_3$

(3) $CH_3-CH_2-\overset{CH_3}{\overset{|}{C}}=CH_2$ (4) $CH_3-CH_2CH=CH-CH_3$

Ans. (2)

Sol.



36. The process that is NOT endothermic in nature is :

(1) $H(g) + e^- \rightarrow H^-(g)$ (2) $Ar(g) + e^- \rightarrow Ar^-(g)$ (3) $O^-(g) + e^- \rightarrow O^{2-}(g)$ (4) $Na(g) \rightarrow Na^+(g) + e^-$

Ans. (1)

Sol. $H(g) + e^- \xrightarrow{\text{endothermic}} H^-(g)$ $\quad \Delta H_{eg} = -72\ KJ/mol$

$O^-(g) + e^- \xrightarrow{\text{endothermic}} O^{2-}(g)$ $\quad \Delta H_{eg} = +744\ KJ/mol$

$Ar(g) + e^- \xrightarrow{\text{endothermic}} Ar^-(g)$ $\quad \Delta H_{eg} = +96\ KJ/mol$

$Na(g) \xrightarrow{\text{endothermic}} Na^+(g) + e^-$ $\quad IE = 495.8\ KJ/mol$

37. The mechanism of action of "Terfenadine" (Seldane) is :

(1) Helps in the secretion of histamine (2) Inhibits the secretion of histamine

(3) Activates the histamine receptor (4) Inhibits the action of histamine receptor

Ans. (4)

Sol. Seldane act as antihistamines and interfere with the natural action of histamine by competing with histamine for binding sites of receptor.

38. 250 mL of a waste solution obtained from the workshop of a goldsmith contains 0.1 M $AgNO_3$ and 0.1 M AuCl. The solution was electrolyzed at 2 V by passing a current of 1 A for 15 minutes. The metal/metals electrodeposited will be : $\left(E^o_{Ag^+/Ag} = 0.80V, E^o_{Au^+/Au} = 1.69V\right)$

(1) silver and gold in proportion to their atomic weights

(2) silver and gold in equal mass proportion

(3) only silver

(4) only gold

Ans. (4){NTA answer given is (1)}

Sol. $\text{Charge}(q) = \frac{it}{96500}F = \frac{1 \times 15 \times 60}{96500} = \frac{900}{96500} = \frac{9}{965}F = 0.0093\ F$

No. of moles of Au^+ = 0.025 & No. of moles of Ag^+ = 0.025

Species with higher value of SRP will get deposited first at cathode.

(i) $Au^{+}(aqs) + e^{-} \longrightarrow Au(s)$
0.025 0.0093 mol

so only Au will get deposited.

39. The processes of calcination and roasting in metallurgical industries, respectively, can lead to :

(1) Photochemical smog and ozone layer depletion

(2) Photochemical smog and global warming

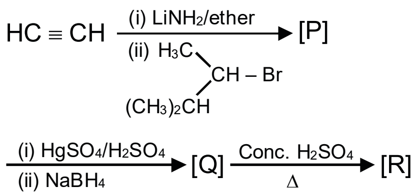
(3) Global warming and acid rain

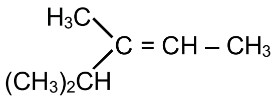
(4) Global warming and photochemical smog

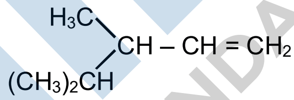
Ans. (3)

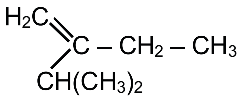
Sol. In Calcination and roasting CO_2 and SO_2 are released which are responsible for Global warning and acid rain.

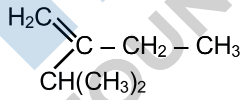
40. The major product [R] in the following sequence of reaction is :



(1) 

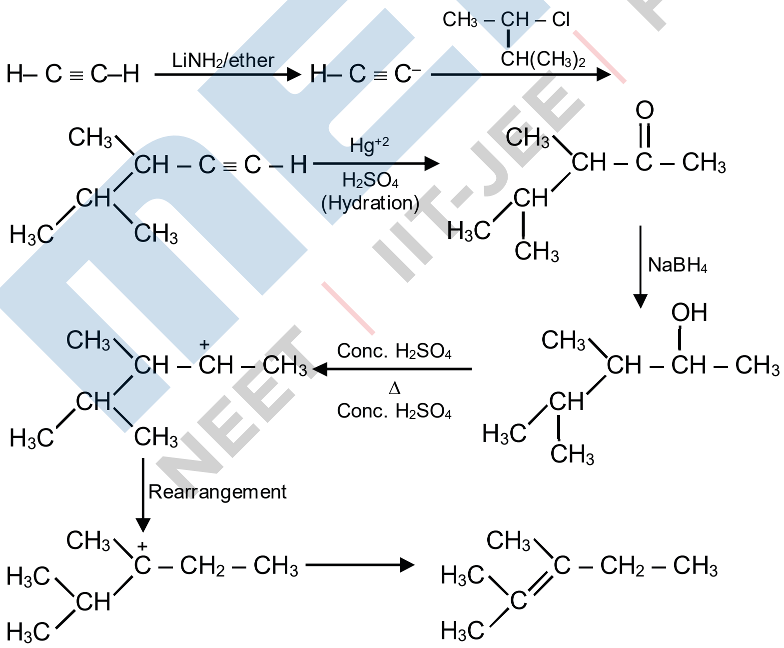
(2) 

(3) 

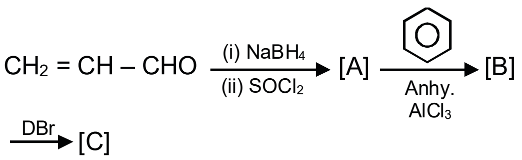
(4) 

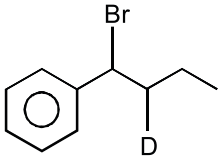
Ans. (4)

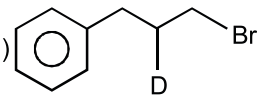
Sol.

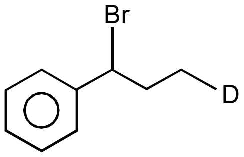


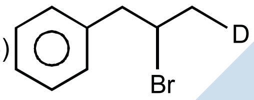
41. The major product [C] of the following reaction sequence will be :



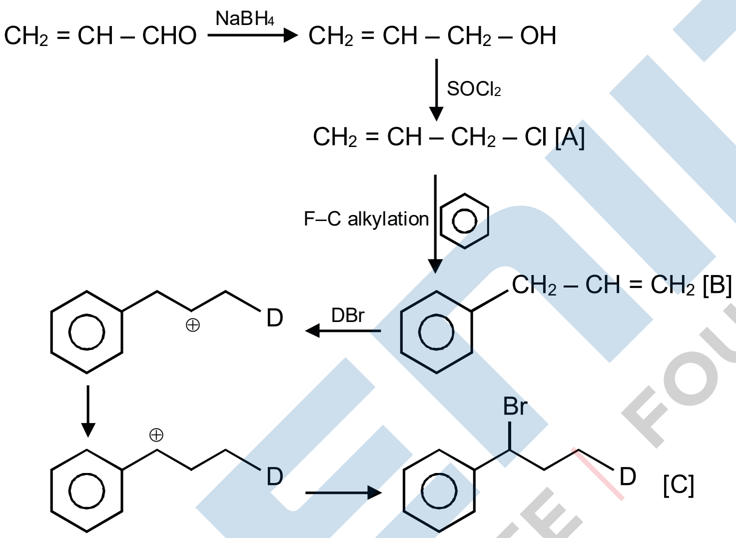
(1) 

(2) 

(3) 

(4) 

Ans. (3)

Sol. 

42. Five moles of an ideal gas at 1 bar and 298 K is expanded into vacuum to double the volume. The work done is :

(1) $-RT(V_2 - V_1)$ (2) Zero (3) $-RT \text{ in } V_2 / V_1$ (4) $C_2(T_2 - T_1)$

Ans. (2)

Sol. $W = -P_{ext}\,\Delta V$

In expansion against vacuum $P_{ext} = 0$

So work done is zero.

43. A sample of red ink (a colloidal suspension) is prepared by mixing eosine dye, egg white, HCHO and water. The component which ensures stability of the ink sample is :

(1) HCHO (2) Water (3) Eosine dye (4) Egg white

Ans. (4)

Sol. Blue ink is a colloidal sol, so it can be stabilised by material like natural gum or Egg white/albumen.

44. The shortest wavelength of H atom in the Lyman series is λ_1. The longest wavelength in the Balmer series of He+ is :

(1) $\frac{9\lambda_1}{5}$ (2) $\frac{36\lambda_1}{5}$ (3) $\frac{5\lambda_1}{9}$ (4) $\frac{27\lambda_1}{5}$

Ans. (1)

Sol. For hydrogen atom :

For Lyman series $n_1 = 1$ & $n_2 = \infty$

$\frac{1}{\lambda_H} = R_H\left[\frac{1}{1} - \frac{1}{\infty}\right]$ So, $\lambda = \frac{1}{R_H}$

For He^+ ion

Balmer series $n_1 = 2$ & $n_2 = 3$

$$\frac{1}{\lambda_{He^+}} = R_H \times Z^2\left[\frac{1}{4} - \frac{1}{9}\right]$$

$$\frac{1}{\lambda_{He^+}} = R_H \times 4 \times \frac{5}{36}$$

$$\frac{1}{\lambda_{He^+}} = \frac{5}{9}R_H = \left(\frac{5}{9}\right)\frac{1}{\lambda}$$

$$\lambda_{He^+} = \frac{9}{5}\lambda$$

45. The Crystal Field Stabilization Energy (CFSE) of $[CoF_3(H_2O)_3]$ ($\Delta_0 < P$) is :

(1) $-0.4\ \Delta_0$ (2) $-0.8\ \Delta_0$ (3) $-0.4\ \Delta_0 + P$ (4) $-0.8\ \Delta_0 + 2P$

Ans. (1)

Sol. $[Co(H_2O)_3F_3]$ $Co^{3+} = 3d_6 4s_0 \Rightarrow t_{2g}^{2,1,1}, e_g^{1,1}$

CFSE $= [-0.4n t_{2g} + 0.6n_{eg}]\Delta_0 + n(P)$

$= [-0.4 \times 4 + 0.6 \times 2]\Delta_0 + 0$

$= -0.4\Delta_0$

SECTION – 2 : (Maximum Marks : 20)

This section contains FIVE (05) questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.

If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Full Marks : +4 If ONLY the correct option is chosen.

Zero Marks : 0 In all other cases

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46. The number of molecules with energy greater than the threshold energy for a reaction increases five fold by a rise of temperature from 27° C to 42° C. Its energy of activation in J/mol is …………..

(Take in 5 = 1.6094; R = 8.314 J mol^{-1})

Ans. (84297)

Sol. $k = Ae^{-\frac{Ea}{RT}}$

$$\ln\left(\frac{K_2}{K_1}\right) = \frac{Ea}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\ln(5) = \frac{Ea}{8.314}\left[\frac{1}{300} - \frac{1}{315}\right]$$

$$1.6094 = \frac{Ea}{8.314}\left[\frac{15}{300 \times 315}\right]$$

Ea = 84297J

47. The osmotic pressure of a solution of NaCl is 0.10 atm and that of a glucose solution is 0.20 atm. The osmotic pressure of a solution formed by mixing 1 L of the sodium chloride solution with 2 L of the glucose solution is x × 10^{-3} atm. x is ………….. (nearest integer)

Ans. (167)

Sol. $\Pi = i\,CRT = i\left[\frac{n}{V}\right]RT$

$$\Pi_{final} = \frac{(\pi_1 V_1) + (\pi_2 V_2)}{V_1 + V_2}$$

$$\Pi_{final} = \frac{(0.1 \times 1) + (0.2 \times 2)}{3}$$

$$= \frac{(0.1 + 0.4)}{3} = \frac{0.5}{3} = \frac{500}{3} \times 10^{-3}\text{ atm}$$

so X = 167

48. A 100 mL solution was made by adding 1.43 g of $Na_2CO_3.xH_2O$. The normality of the solution is 0.1 N. The value of x is ………..

(The atomic mass of Na is 23 g/mol)

Ans. (10)

Sol. Equivalent of solute = 0.1 × 0.1

Mol of solute ($Na_2CO_3.xH_2O$) $=[0.1\times 0.1]\frac{1}{2}$

Mass of $Na_2CO_3.xH_2O$ $=[0.1\times 0.1]\frac{1}{2}+[106+18x]=1.43$

$\Rightarrow$ [106 + 18x = 286

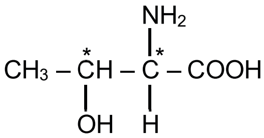
18x = 180

x = 10

49. The number of chiral centres present in threonine is …………

Ans. (2)

Sol.



Threonine have two chiral carbon atom.

50. Consider the following equations :

$2Fe^{2+} + H_2O_2 \longrightarrow xA + yB$ (in basic medium)

$2MnO_4^{-} + 6H^+ + 5H_2O_2 \longrightarrow x'C + y'D + z'E$

(in acidic medium)

The sum of the stoichiometric coefficients x, y, x', y' and z' for products A, B, C, D and E, respectively, is ………..

Ans. (19)

Sol. (i) $2Fe^{2+} + H_2O_2 \longrightarrow 2Fe^{3+} + 2OH^-$

(ii) $2MnO_4^{-} + 5H_2O_2 + 6H^+ \longrightarrow 2Mn^{2+} + 5O_2 + 8H_2O$

So sum of $(x + y + x^1 + y^1 + z^1) = 2 + 2 + 2 + 5 + 8 = 19$

PART-C : MATHEMATICS

SECTION – 1 : (Maximum Marks : 80)

Single Choice Type

This section contains 20 Single choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

Full Marks : +4 If ONLY the correct option is chosen.

Negative Marks : –1 (minus one) mark will be deducted for indicating incorrect response.

51. Let f : (0, ∞) → (0, ∞) be a differentiable function such that f(1) = e and $\lim_{t\to x}\frac{t^2f^2(x)-x^2f^2(t)}{t-x}=0$. If f(x) = 1, then x is equal to :

(1) $\frac{1}{e}$ (2) 2e (3) $\frac{1}{2e}$ (4) e

Ans. (1)

Sol. $\lim_{t\to x}\frac{t^2f^2(x)-x^2f^2(t)}{t-x}=0$

using L'Hospital

$\lim_{t\to x}\frac{2tf^2(x)-x^2 2f(t)f'(t)}{1}=0$

x^2 2f(x) f'(x) – 2x $f^2(x)$ = 0

2x f(x) [xf'(x) – f(x)] = 0

f(x) ≠ 0 so xf'(x) = f(x)

$x\frac{dy}{dx}=y$

$\frac{1}{y}dy=\frac{1}{x}dx$

Integration ℓny = ℓnx + ℓnc

y = cx ⇒ f(x) = cx

Now f(1) = c = e

so f(x) = ex

now f(x) = 1

ex = 1 ⇒ $x=\frac{1}{e}$

52. Contrapositive o the statement :

'If a function f is differentiable at a, then it is also continuous at a', is :

(1) If a function f is not continuous at a, then it is not differentiable at a.

(2) If a function f is continuous at a, then it is differentiable at a.

(3) If a function f is not continuous at a. then it is differentiable at a.

(4) If a function f is continuous at a, then it is not differentiable at a.

Ans. (1)

Sol. Contrapositive of $p \Rightarrow q$ is $\sim q \Rightarrow \sim p$

53. The solution of the differential equation $\frac{dy}{dx} = \frac{y+3x}{\log_e(y+3x)} + 3 = 0$ is

(where C is a constant of integration)

(1) $x - 2\log_e(y + 3x) = C$

(2) $x - \log_e(y + 3x) = C$

(3) $y + 3x - \frac{1}{2}(\log_e x)^2 = C$

(4) $y - \frac{1}{2}(\log_e(y+3x))^2 = C$

Ans. (4)

Sol. $\frac{dy}{dx} - \frac{y+3x}{\ln(y+3x)} + 3 = 0$

$\frac{dy}{dx} + 3 = \frac{y+3x}{\ln(y+3x)}$

$\frac{d}{dx}(y+3x) = \frac{y+3x}{\ln(y+3x)}$

$\int \frac{\ell n(y+3x)}{(y+3x)} d(y+3x) = \int dx$

Let $\ln(y + 3x) = t$

$\frac{1}{(y+3x)} d(y+3x) = dt$

$\int tdt = \int dx$

$\frac{t^2}{2} = x + c$

$\frac{(\ell n(y+3x))^2}{2} = x + c$

54. If for some positive integer n, the coefficients of three consecutive terms in the binomial expansion of $(1 + x)^{n+5}$ are in the ratio 5 : 10 : 14, then the largest coefficient in the expansion is :

(1) 330 (2) 252 (3) 792 (4) 462

Ans. (4)

Sol. Let three consecutive term are T_r, T_{r+1}, T_{r+2}

Hence $\frac{T_r}{T_{r+1}} = \frac{5}{10}$ and $\frac{T_{r+1}}{T_{r+2}} = \frac{10}{14}$

$\frac{T_{r+1}}{T_r} = 2$ $\qquad$ $\frac{{}^{n+5}C_r}{{}^{n+5}C_{r+1}} = \frac{5}{7}$

$\frac{{}^{n+5}C_r}{{}^{n+5}C_{r-1}} = 2$ $\qquad$ $\frac{{}^{n+5}C_{r+1}}{{}^{n+5}C_r} = \frac{7}{5}$

$\frac{(n+5)-r+1}{r} = 2$ $\qquad$ $\frac{(n+5)-(r+1)+1}{r+1} = \frac{7}{5}$

$n - r + 6 = 2r$ $\qquad \frac{n-r+5}{r+1} = \frac{7}{5}$

$n - 3r + 6 = 0$ ....(i)

$5n - 5r + 25 = 7r + 7$

$5n - 12r + 18 = 0$ ....(ii)

Multiply equation (i) by 5

$5n - 15r + 30 = 0$

$5n - 12r + 18 = 0$

$- \quad + \quad -$

$-3r + 12 = 0 \Rightarrow r = 4, n = 6$

hence greatest coefficient will be of middle term = ${}^{n+5}C_5 = {}^{11}C_5 = 462$

55. The circle passing through the intersection of the circles, $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 - 4y = 0$, having its centre on the line, $2x - 3y + 12 = 0$, also passes through the point :

(1) (1, –3) (2) (–1, 3) (3) (–3, 6) (4) (–3, 1)

Ans. (3)

Sol. By family of circle, passing through intersection of given circle will be member of

$S_1 + \lambda S_2 = 0$ family ($\lambda \neq 1$)

$(x^2 + y^2 - 6x) + \lambda(x^2 + y^2 - 4y) = 0$

$(\lambda + 1)x^2 + (\lambda + 1)y^2 - 6x - 4\lambda y = 0$

$$x^2 + y^2 - \frac{6}{\lambda+1}x - \frac{4\lambda}{\lambda+1}y = 0$$

Centre $\left(\frac{3}{\lambda+1}, \frac{2\lambda}{\lambda+1}\right)$

centre lies on $2x - 3y + 12 = 0$

$$2\left(\frac{3}{\lambda+1}\right) - 3\left(\frac{2\lambda}{\lambda+1}\right) + 12 = 0$$

$6\lambda + 18 = 0$

$\lambda = -3$

Circle $x^2 + y^2 + 3x - 6y = 0$

56. In a game two players A and B take turns in throwing a pair of fair dice starting with player A and total of scores on the two dice, in each throw is noted. A wins the game if he throws a total of 6 before B throws a total of 7 and B wins the game if he throws a total of 7 before A throws a total of six. The game stops as soon as either of the players wins. The probability of A winning the game is :

(1) $\frac{30}{61}$ (2) $\frac{5}{6}$ (3) $\frac{5}{31}$ (4) $\frac{31}{61}$

Ans. (1)

Sol. sum 6 $\rightarrow$ (1, 5), (5, 1), (3, 3), (2, 4), (4, 2)

sum 4 $\rightarrow$ (1, 6), (6, 1), (5, 2), (2, 5), (3, 4), (4, 3)

$P(A \text{ wins}) = P(A) + P(\bar{A}).P(\bar{B}).P(A) + P(\bar{A})P(\bar{B}).P(\bar{A}).P(\bar{B}).P(A) + \ldots$

this is infinite G.P. with common ratio P(A) × P(B)

Probability of A wins $= \dfrac{P(A)}{1 - P(\bar{A})\,P(\bar{B})}$

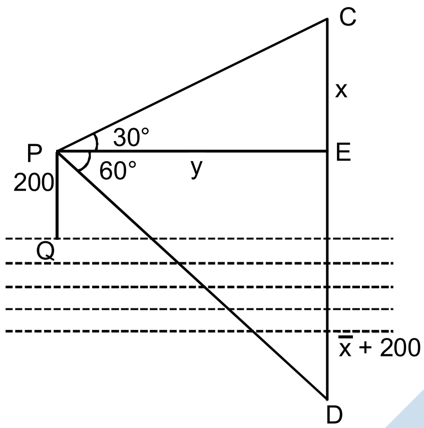
$$= \dfrac{\frac{5}{36}}{1 - \frac{31}{36}\cdot\frac{30}{36}} = \dfrac{30}{61}$$

57. The angle of elevation of a cloud C from a point P, 200 m above a still take is 30°. If the angle of depression of the image of C in the lake from the point P is 60°, then PC (in m) is equal to

(1) 100 (2) $400\sqrt{3}$ (3) $200\sqrt{3}$ (4) 400

Ans. (4)

Sol.



$\tan 30° = \dfrac{x}{y} = \dfrac{1}{\sqrt{3}} \Rightarrow y = \sqrt{3}x$ and $\tan 60° = \dfrac{x + 400}{y}$

x + 400 = 3x

2x = 400, x = 200

$\sin 30° = \dfrac{200}{PC} \Rightarrow PC = 400$

58. The function $f(x) = \begin{cases} \dfrac{\pi}{4} + \tan^{-1}x, & |x| \le 1 \\ \dfrac{1}{2}(|x| - 1), & |x| > 1 \end{cases}$ is :

(1) continuous on R – {–1} and differentiable on R – {–1, 1}.

(2) both continuous and differentiable on R – {–1}.

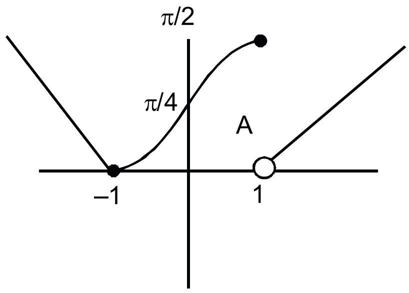
(3) both continuous and differentiable on R – {1}

(4) continuous on R – {1} and differentiable on R – {–1, 1}.

Ans. (4)

Sol. $f(x) = \begin{cases} \dfrac{\pi}{4} + \tan^{-1}x, & |x| \le 1 \\ \dfrac{1}{2}(|x| - 1), & |x| > 1 \end{cases}$:

Graph of f(x) is



f(x) is continuous on R – {1}

f(x) is differentiable on R – {–1,1}

59. Suppose the vectors x_1, x_2 and x_3 are the solutions of the system of linear equations, Ax = b when the vector b on the right side is equal to b_1, b_2 and b_3 respectively. If

$x_1 = \begin{bmatrix}1\\1\\1\end{bmatrix}, x_2 = \begin{bmatrix}0\\2\\1\end{bmatrix}, x_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}, b_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}, b_2 = \begin{bmatrix}0\\2\\0\end{bmatrix}$ and $b_3 = \begin{bmatrix}0\\0\\2\end{bmatrix}$, then the determinant of A is equal of A is equal to :

(1) $\frac{3}{2}$ (2) 4 (3) $\frac{1}{2}$ (4) 2

Ans. (4)

Sol. Let $A = \begin{bmatrix}\alpha_1 & \alpha_2 & \alpha_3\\ \beta_1 & \beta_2 & \beta_3\\ \gamma_1 & \gamma_2 & \gamma_3\end{bmatrix}$

$Ax_1 = b_1 \Rightarrow \begin{bmatrix}\alpha_1 & \alpha_2 & \alpha_3\\ \beta_1 & \beta_2 & \beta_3\\ \gamma_1 & \gamma_2 & \gamma_3\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix}$

$\alpha_1 + \alpha_2 + \alpha_3 = 1$

$\beta_1 + \beta_2 + \beta_3 = 0$

$\gamma_1 + \gamma_2 + \gamma_3 = 0$

similar $2\alpha_2 + \alpha_3 = 0$ and $\alpha_3 = 0$

$2\beta_2 + \beta_3 = 2$ $\quad \beta_3 = 0$

$2\gamma_2 + \gamma_3 = 0$ $\quad \gamma_3 = 2$

$\therefore \alpha_2 = 0, \beta_2 = 1, \gamma_2 = -1,$

$\alpha_1 = 1, \beta_1 = -1, \gamma_1 = -1$

$A = \begin{bmatrix}1 & 0 & 0\\ -1 & 1 & 0\\ -1 & -1 & 2\end{bmatrix} \therefore |A| = 2$

60. Let $a_1, a_2, \ldots.$ an be a given A.P. whose common difference is an integer and $S_n = a_1 + a_2 + \ldots + a_n$. If $a_1 = 1$, $a_1 = 300$ and $15 \le n \le 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to :

(1) (2490, 248) (2) (2490, 249) (3) (2480, 249) (4) (2480, 248)

Ans. (1)

Sol. an = a1 + (n – 1)d ⇒ 300 = 1 + (n – 1)d

$\Rightarrow \quad d = \frac{299}{(n-1)} = \frac{13\times 23}{(n-1)} = \text{integer}$

So n – 1 = ±13, ±23, ±299, ±1

⇒ n = 14, –12, 24, –22, 300, –298, 2, 0

But n ∈ [15, 50] ⇒ n = 24 ⇒ d = 13

Hence $S_{n-4} = S_{20} = \frac{20}{2}[2(1)+(20-1)(13)] = 10[2+247] = 2490$

$a_{n-4} = a_{20} = a_1 + 19d$

= 1 + 19 × 13

= 1 + 247

= 248

61. Let $\bigcup_{i=1}^{50} X = \bigcup_{i=1}^{n} Y_i = T$, where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets X_i's and exactly 6 of sets Y_i's then n is equal to:

(1) 45 (2) 15 (3) 30 (4) 50

Ans. (3)

Sol. $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = Z \quad \therefore \quad \frac{10\times 50}{20} = \frac{5n}{6} \quad \Rightarrow \quad n = 30$

62. The area (in sq. units) of the largest rectangle ABCD whose vertices A and B lie on the x-axis and vertices C and D lie on the parabola, $y = x^2 - 1$ below the x-axis, is :

(1) $\frac{1}{3\sqrt{3}}$ (2) $\frac{4}{3}$ (3) $\frac{4}{3\sqrt{3}}$ (4) $\frac{2}{3\sqrt{3}}$

Ans. (3)

Sol. A (α, 0), β(–α, 0)

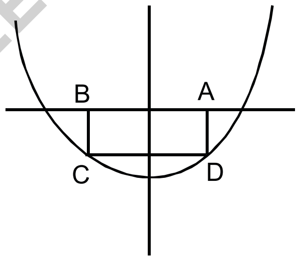
⇒ $D(\alpha, \alpha^2-1)$

Area (ABCD) = (AB) (AD)

⇒ $S = (2\alpha)(1-\alpha 2) = 2\alpha - 2\alpha^3$

$\frac{ds}{d\alpha} = 2 - 6\alpha^2 = 0 \Rightarrow \alpha^2 = \frac{1}{3} \Rightarrow \alpha^2 = \frac{1}{\sqrt{3}}$

Area $= 2\alpha - 2\alpha^3 = \frac{2}{\sqrt{3}} - \frac{2}{3\sqrt{3}} = \frac{4}{3\sqrt{3}}$



63. Let x = 4 be a directrix to an ellipse whose centre is at the origin and its eccentricity is $\frac{1}{2}$. If P(1, β), β > 0 is a point on this ellipse, then the equation of the normal to it at P is

(1) 7x– 4y = 1 (2) 4x – 2y = 1 (3) 8x – 2y = 5 (4) 4x – 3y = 2

Ans. (2)

Sol. $\frac{a}{e} = 4 \Rightarrow a = 4e \Rightarrow a = 2$

$b^2 = a^2(1 - e^2) = 3$

$(1, \beta)$ lies on $\frac{x^2}{4} + \frac{y^2}{3} = 1 \Rightarrow \frac{1}{4} + \frac{\beta^2}{3} = 1 \quad \Rightarrow \quad \beta^2 = \frac{9}{4} \Rightarrow \beta = \frac{3}{2} (\because \beta > 0)$

Normal at$(1, \beta) \Rightarrow \frac{a^2 x}{1} - \frac{b^2 y}{\beta} = a^2 - b^2 \quad \Rightarrow 4x - \frac{3y}{\beta} = 1$

so equation of normal is $4x - 2y = 1$

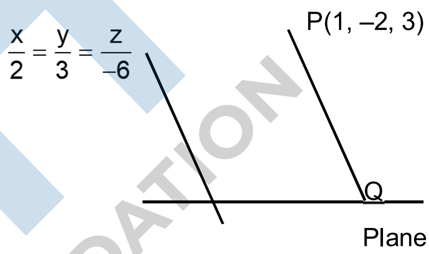
64. The distance of the point (1, –2, 3) from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is :

(1) $\frac{1}{7}$ (2) 7 (3) 1 (4) $\frac{7}{5}$

Ans. (3)

Sol. Equation PQ

$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda$



Let $Q \equiv (2\lambda + 1, 3\lambda - 2, -6\lambda + 3)$

Q lies on $x - y + z = 5$

$\Rightarrow (2\lambda + 1) - (3\lambda - 2) + (-6\lambda + 3) \Rightarrow \lambda = \frac{1}{7} \Rightarrow Q \equiv \left(\frac{9}{7}, \frac{11}{7}, \frac{15}{7}\right)$

$PQ = \sqrt{\left(1 - \frac{9}{7}\right)^2 + \left(-2 + \frac{11}{7}\right)^2 + \left(3 - \frac{15}{7}\right)^2} = 1$

65. If the perpendicular bisector of the line segment joining the points P(1, 4) and Q(k, 3) has y-intercept equal to –4, then a value of k is ;

(1) $\sqrt{15}$ (2) –4 (3) –2 (4) $\sqrt{14}$

Ans. (2)

Sol. Mid point PQ $\left(\frac{k+1}{2}, \frac{7}{2}\right)$

and slope of PQ $= \frac{1}{1-k}$

so equation of perpendicular bisector of PQ

$y - \frac{7}{2} = (k-1)\left(x - \frac{k+1}{2}\right) \quad \ldots\ldots\ldots(1)$

Now it's y intercept = – 4

so equation (1) satisfy (0, –4)

$\Rightarrow -\frac{15}{2} = -\left(\frac{k^2 - 1}{2}\right)$

$k^2 = 16 \Rightarrow k = 4$

66. The integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan^3 x \cdot \sin^2 3x(2\sec^2 x\sin^2 3x + 3\tan x\sin 6x)\,dx$ is equal to :

(1) $-\frac{1}{9}$ (2) $\frac{9}{2}$ (3) $-\frac{1}{18}$ (4) $\frac{7}{18}$

Ans. (3)

Sol. $\int_{\pi/6}^{\pi/3}\left(\frac{\frac{d}{dx}(\tan^4 x)}{2}\cdot\sin^4 3x + \tan^4 x\cdot\frac{\frac{d}{dx}(\sin^4 3x)}{2}\right) = \frac{1}{2}\int_{\pi/6}^{\pi/3}\frac{d}{dx}(\tan^4 x.\sin^4 3x)\,dx$

$= \frac{1}{2}\left[\tan^4 x.\sin^4 3x\right]_{\pi/6}^{\pi/3} = \frac{1}{2}\cdot\left[(3)^4\times 0 - \frac{1}{\left(\sqrt{3}\right)^4}\right] = -\frac{1}{2}\times\frac{1}{9} = -\frac{1}{18}$

67. The minimum value of 2sinx + 2cosx is :

(1) $2^{-1+\frac{1}{\sqrt{2}}}$ (2) $2^{-1+\sqrt{2}}$ (3) $2^{1-\sqrt{2}}$ (4) $2^{1-\frac{1}{\sqrt{2}}}$

Ans. (4)

Sol. Using A.M. ≥ G.M.

$\frac{2^{\sin x}+2^{\cos x}}{2} \geq \sqrt{2^{\sin x}\cdot 2^{\cos x}}$

$\frac{2^{\sin x}+2^{\cos x}}{2} \geq 2^{\frac{\sin x+\cos x}{2}}$ ...(i)

Now $-\sqrt{2} \leq \sin x + \cos x = \sqrt{2}$

So $-\frac{1}{\sqrt{2}} \leq \frac{\sin x+\cos x}{2} \leq \frac{1}{\sqrt{2}}$

minimum value of $2^{\frac{\sin x+\cos x}{2}} = 2^{-\frac{1}{\sqrt{2}}}$

so by (i)

minimum value of $\frac{2^{\sin x}+2^{\cos x}}{2} = 2^{-\frac{1}{\sqrt{2}}}$

minimum value of $2^{\sin x}+2^{\cos x} = 2^1\cdot 2^{-\frac{1}{\sqrt{2}}} = 2^{1-\frac{1}{\sqrt{2}}}$

68. If a and b are real numbers such that $(2 + \alpha)^4 = a + b\alpha$, where $\alpha = \frac{-1+i\sqrt{3}}{2}$ then a + b is equal to :

(1) 33 (2) 24 (3) 9 (4) 57

Ans. (3)

Sol. $(2 + \alpha)^4 = a + b\alpha$

$(4 + \alpha^2 + 4\alpha)^2 = a + b\alpha$ ∵ $1 + \alpha = -\alpha^2$

$9\alpha^4 = a + b\alpha$

$9\alpha = a + b\alpha \Rightarrow a = 0, b = 9 \Rightarrow a + b = 9$

69. Let $\lambda \neq 0$ be in R. If α and β are the roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation, $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to :

(1) 27 (2) 36 (3) 9 (4) 18

Ans. (4)

Sol. Given $3\alpha^2 - 10\alpha + 27\lambda = 0$ ....(i)

$3\alpha^2 - 3\alpha + 6\lambda = 0$ ....(ii)

subtract $-7\alpha + 21\lambda = 0$

$3\lambda = \alpha$

by (ii) $9\lambda^2 - 3\lambda + 2\lambda = 0$

$\Rightarrow \lambda = 0, \frac{1}{9}$

$\therefore$ given equation are $x^2 - x + \frac{2}{9} = 0$ and $3x^2 - 10x + 3 = 0$

$\therefore \alpha = \frac{1}{3}, \beta = \frac{2}{3}, \alpha = \frac{1}{3}, \gamma = 3$

$\therefore \frac{\beta\gamma}{\lambda} = \frac{\frac{2}{3}\cdot 3}{\frac{1}{9}} = 18$

70. If the system of equations

$x + y + z = 2$

$2x + 4y - z = 6$

$3x + 2y + \lambda z = \mu$

(1) $\lambda + 2\mu = 14$ (2) $2\lambda - \mu = 5$ (3) $2\lambda + \mu = 14$ (4) $\lambda - 2\mu = -5$

Ans. (3)

Sol. $D = 0 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 3 & 2 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = \frac{9}{2}p$

$D_3 = 0 \Rightarrow \begin{vmatrix} 1 & 1 & 2 \\ 2 & 4 & 6 \\ 3 & 2 & \mu \end{vmatrix} = 0 \Rightarrow \mu = 5$

SECTION – 2 : (Maximum Marks : 20)

This section contains FIVE (05) questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.

If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Full Marks : +4 If ONLY the correct option is chosen.

Zero Marks : 0 In all other cases

71. Let PQ be a diameter of the circle $x^2 + y^2 = 9$. If α and β are the lengths of the perpendiculars from P and Q on the straight line, x + y = 2 respectively, then the maximum value of $\alpha\beta$ is ……

Ans. 7

Sol. Let P(3cosθ, 3sinθ) ∴ Q (–3 cosθ, –3sinθ)

given line x + y – 2 = 0

$$\therefore \alpha = \frac{|3\cos\theta + 3\sin\theta - 2|}{\sqrt{2}}$$

$$\beta = \frac{|-3\cos\theta - 3\sin\theta - 2|}{\sqrt{2}}$$

$$\therefore \alpha\beta = \left|\frac{(3\cos\theta + 3\sin\theta - 2)\cdot(3\cos\theta + 3\sin\theta + 2)}{2}\right| = \left|\frac{9(1+\sin 2\theta) - 4}{2}\right|$$

∴ maximum $\alpha\beta = 7$

72. If the variance of the following frequency distribution :

Class :	10-20	20-30	30-40
Frequency :	2	x	2

is 50, then x is equal to …..

Ans. 4

Sol.

x_i	15	25	35
f_i	2	x	2

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{30 + 25x + 70}{4 + x} = 25$$

$$\sigma^2 = 50 = \frac{\Sigma f_i x_i^2}{\Sigma f_i} - (\bar{x})^2$$

$$50 = \frac{450 + 625x + 2450}{4 + x} - (25)^2$$

$$50 = \frac{2900 + 625x}{4 + x} - 625 \Rightarrow 675(4 + x) = 2900 + 625x \Rightarrow 50x = 200 \Rightarrow x = 4$$

73. A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four of the answers are correct, is ……

Ans. 135

Sol. No. of ways of giving wrong answer = 3

required no. of ways $= {}^6C_4\,(1)^4 \times (3)^2$

$= 15(9) = 135$

74. Let {x} and [x] denote the fractional part of x and the greatest integer $\le$ x respectively of a real number x. if $\int_0^n \{x\}dx, \int_0^n [x]dx$ and $10(n^2 - n)$, $(n \in N, n > 1)$ are three consecutive terms of a G.P. then n is equal to ……

Ans. 21

Sol. $\int_0^n \{x\}dx = n\int_0^1 x\,dx = n\left(\frac{x^2}{2}\right)_0^1 = \frac{n}{2}$

and $\int_0^n [x]dx = \int_0^n (x - \{x\})dx = n\left(\frac{x^2}{2}\right)_0^1 = \frac{n}{2}$

now $\frac{n}{2}, \frac{n^2 - n}{2}$ and $10(n^2 - n)$ are in Geometric progression

$= \left(\frac{n^2 - n}{2}\right)^2 = \frac{n}{2}\cdot 10(n^2 - n)$

$\Rightarrow \frac{n^2(n-1)^2}{4} = +5\cdot n^2(n-1)$

$\Rightarrow n - 1 = 20 \quad \Rightarrow \quad n = 21$

75. IF $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$, then the value of $\left|\hat{i}\times(\vec{a}\times\hat{i})\right|^2 + \left|\hat{j}\times(\vec{a}\times\hat{j})\right|^2 + \left|\hat{k}\times(\vec{a}\times\hat{k})\right|^2$ is equal to :

Ans. 18.

Sol. Let $\vec{a} = x\hat{i} = x\hat{i} + y\hat{j} + z\hat{k}$

$\hat{i}\times\left(\vec{a}\times\hat{i}\right) = \left(\hat{i}.\hat{i}\right)\vec{a} - \left(\vec{a}.\hat{i}\right)\hat{i} = y\hat{j} + z\hat{k}$

similarly $\hat{j}\times\left(\vec{a}\times\hat{j}\right) = x\hat{i} + z\hat{k}$ and $\hat{k}\times\left(\vec{a}\times\hat{k}\right) = x\hat{i} + y\hat{k}$

$\left|\hat{i}\times\left(\vec{a}\times\hat{i}\right)\right|^2 + \left|\hat{j}\times\left(\vec{a}\times\hat{j}\right)\right|^2 + \left|\hat{k}\times\left(\vec{a}\times\hat{k}\right)\right|^2$

$\left|y\hat{j} + z\hat{k}\right|^2 + \left|x\hat{i} + z\hat{k}\right|^2 + \left|x\hat{i} + y\hat{j}\right|^2 = 2|a|^2 = 2(9) = 18$